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The nonstationary Casimir effect in a cavity with periodical time-dependent conductivity of a semiconductor mirror

V V Dodonov[1] **and A V Dodonov**[2]

[1] Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70910-900 Brasília, DF, Brazil
[2] Departamento de Física, Universidade Federal de São Carlos, Via Washington Luiz Km 235, São Carlos, 13565-905, SP, Brazil

E-mail: vdodonov@fis.unb.br and adodonov@df.ufscar.br

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Abstract

We consider the problem of photon creation from vacuum or thermal states inside a cavity with periodical time-dependent conductivity of a thin semiconductor boundary layer, simulating periodical displacements of the wall. Our approach is based on the consistent model of a quantum-damped harmonic oscillator with arbitrary time-dependent frequency and damping coefficients in the framework of the Heisenberg–Langevin equations with two noncommuting delta-correlated noise operators. We calculate the rate of photon generation under the resonance conditions, taking into account the internal dissipation inside the semiconductor. This rate depends mainly on two parameters: the total intensity of the laser pulse and the recombination time of photo-excited carriers in the semiconductor slab (for fixed mobility of carriers and geometry). Optimal values of these parameters and dimensions of the cavity are found for the TE and TM modes. The influence of temperature and detuning from strict resonance is analysed.

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1. Introduction

After Moore's paper [1], the fascinating effect of photon creation from vacuum due to the motion of boundaries was the subject of many theoretical studies (extensive lists of publications can be found in [2, 3]). Since this is a relativistic effect of the second order, its experimental verification is a very difficult problem. For this reason, many authors studied the conditions of *resonance amplification* of the vacuum fluctuations in *cavities with oscillating walls* [4–13]. The theoretical prediction is that under the conditions of strict resonance, the number of photons created from the vacuum state in the selected mode of an closed ideal cavity is given by the typical parametric resonance formula

$$\mathcal{N}_n^{(\mathrm{res})} = \sinh^2(n\nu), \tag{1}$$

where n is the number of oscillations made by the walls, and the small coefficient ν is proportional to the amplitude value of the relative shift of the mode eigenfrequency due to the change $\Delta L(t)$ of the distance L between the walls. Very roughly, $\nu \sim \max|\Delta L/L|$. If the necessary conditions can be maintained during the time corresponding to $n > n_c \sim \nu^{-1}$ oscillations, then $\mathcal{N}_n$ exceeds the unit value and grows exponentially with the further increase of n, so the photons are really created and can be detected using various schemes.

However, it is still unclear how to excite oscillations of the wall surfaces with high amplitudes at the resonance frequencies in the GHz domain, which correspond to cavities with dimensions of the order of a few centimetres (leaving aside Schwinger's dream [14] of creating visible photons). Since internal stresses inside the oscillating wall are proportional to the square of the frequency, the parameter ν cannot exceed a value of the order 10^{-8} [6]. This extremely small value imposes very hard restrictions on the quality factor of the cavity and the admissible deviations from the exact resonance frequency.

These restrictions can be significantly softened in the scheme of the experiment recently proposed by the group of the University of Padua [15]. The main idea is to use, instead of a real moving metallic surface, an *effective* electron-hole 'plasma mirror', which can be created periodically on the surface of a semiconductor slab by illuminating it with a sequence of powerful femtosecond laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, the highly conducting layer will periodically appear and disappear on the surface of the semiconductor, thus simulating periodical displacements of the boundary. Using semiconductor slabs with thickness of the order of 1 mm, one can hope to obtain the values $\nu \sim |(\Delta L)_{\mathrm{eff}}/L| \sim 10^{-2}$, reducing by several orders of magnitude the number of oscillations of the boundary necessary to produce photons in the cavity, and relaxing the requirements for the Q-factor of the cavity and the admissible detuning from the exact resonance. In particular, one could expect that several hundred oscillations (or, equivalently, laser pulses) would be sufficient to generate a detectable number of microwave quanta in the cavity.

It appears, however, that the simple formula (1) overestimates the number of created photons in the case involved, because it does not take into account inevitable losses inside the semiconductor slab during the excitation–recombination process. This is the immediate consequence of the fact that the dielectric permeability $\epsilon(x)$ of the semiconductor medium is a *complex function*: $\epsilon = \epsilon_1 + \mathrm{i}\epsilon_2$, where $\epsilon_2 = 2\sigma/f_0$, σ and f_0 being the conductivity (in CGS units) and frequency in Hz, respectively. Good conductors have $\epsilon_2 \sim 10^8$ at microwave frequencies, which is essentially bigger than $\epsilon_1 \sim$ 1–10. Although ϵ_2 is negligibly small in the non-excited semiconductor at low temperatures, it rapidly and continuously grows up to the values of the order 10^5–10^6 during the laser pulse (for the maximum concentration of created carriers 10^{18}–10^{19} cm^{-3} and the mobility $b = 3\times 10^6$ CGS units $= 1$ m^2 V^{-1} s^{-1}), returning to zero after the recombination time. Therefore, the standard quantization scheme used in [6, 8, 10–13], based on the assumption of *unitary evolution* of the quantum system, does not work in the case of strongly dissipative 'effective mirrors', and one needs its generalization.

The aim of this paper is to give realistic estimations of the number of photons which could be produced inside the cavity with a semiconductor time-dependent 'mirror' and to find optimal values of different parameters which influence the rate of photon generation, such as the energy and periodicity of laser pulses, the recombination time and the dimensions of the cavity. In section 2, we present the simplest quantum model of the effect and obtain the general formula (11) for the number of created photons, which takes into account the losses and detuning from the strict resonance, generalizing the results obtained recently in [3, 16]. In section 3, we derive a simple approximate analytical formula for the time dependence of the effective complex frequency of the field mode during the process of excitation–recombination in the case of short laser pulses and high absorption coefficient of the semiconductor slab. Surprisingly, this formula does not depend on the field geometry (TE or TM modes), except for some scaling factor. In the concluding section 4, we evaluate the expected values of the photon generation rate and find the optimal values of parameters. We also discuss briefly possible spurious effects, emphasizing the importance of precise knowledge of the resonance periodicity of pulses.

2. The model of a quantum nonstationary damped oscillator

It is well known that the electromagnetic field in a cavity formed by *ideal* mirrors and filled with a dielectric medium *without losses* can be described in terms of an infinite set of *uncoupled* harmonic oscillators (which represent the amplitudes of the normal field modes), provided the geometry is fixed and the properties of the medium do not depend on time. If the boundaries can move (remaining ideal ones) or the parameters of the (lossless) dielectric medium can change with time, then the dynamics of the field can be described in terms of *coupled* oscillators with *time-dependent* frequencies and coupling coefficients, which are determined by the *instantaneous* geometry and properties of the medium [8, 17] (for a complete list of the relevant references, see [2, 3]).

In a generic case, *all* modes interact, but the number of generated quanta in each mode is extremely small in the realistic case of small variations of the parameters. An important exception is the case of *resonance*, when the parameters oscillate with the frequency ω_R, which is close to twice the frequency $2\omega_0$ of some mode. For *harmonic* oscillations of the boundary, it was shown in [6, 8, 11–13] that if the spectrum of unperturbed field eigenmodes does not contain equidistant parts (i.e., ω_R is not close to the difference between some eigenfrequencies), then the interaction between modes is strongly suppressed in the long-time limit, so that its influence on the dynamics of the resonance mode can be neglected. We suppose that the model of a one-dimensional quantum oscillator with a time dependent frequency can be used for *arbitrary periodical* changes of the cavity parameters, if the periodicity of these changes T is close to a multiple of the period of field oscillations in the selected mode $T_0 = 2\pi/\omega_0$. In this case, the parametric excitation can be achieved due to the presence of harmonics of the fundamental exciting frequency $2\pi/T$. However, no harmonics should be close to the difference $|\omega_j - \omega_k|$ between the frequencies of the jth and kth modes, in order to avoid the interaction between these modes.

Considering a cavity with an effective semiconductor time-dependent 'mirror', one must take into account that the 'instantaneous' frequency of the cavity mode is a *complex* function of time $\Omega(t) = \omega(t) - \mathrm{i}\gamma(t)$, where the imaginary part $\gamma(t)$ can be interpreted as the *amplitude-damping coefficient*. Thus, we arrive at the problem of correct description of a quantum-damped oscillator with arbitrary time-dependent frequency and damping coefficient. Since damping implies the appearance of the 'width' of the frequency ω_k of the kth mode, which has an order of γ_k, the condition of absence of resonances between the kth and jth modes and the nth harmonics of the periodical changes of parameters can be written as $|\omega_j - \omega_k - 2\pi n/T| > \gamma_k + \gamma_j$, and we suppose that it is satisfied for all integers $k \neq j$ and n.

We follow the phenomenological 'minimal noise' model, proposed recently in [16]. It is based on the Heisenberg–Langevin equations

$$\mathrm{d}\hat{x}/\mathrm{d}t = \hat{p} - \gamma(t)\hat{x} + \hat{F}_x(t), \qquad \mathrm{d}\hat{p}/\mathrm{d}t = -\gamma(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t), \quad (2)$$

where $\hat{x}$ and $\hat{p}$ are the dimensionless quadrature operators of the selected mode, normalized in such a way that the mean number of photons equals $\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle$ (in particular, we put $\hbar = 1$), whereas $\hat{F}_x(t)$ and $\hat{F}_p(t)$ are *noncommuting delta-correlated noise operators* (but they commute with $\hat{x}$ and $\hat{p}$)

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t), \qquad j,k = x,p, \tag{3}$$

$$\chi_{xp}(t) = -\chi_{px}(t) = \mathrm{i}\gamma(t), \qquad \chi_{pp}(t) = \chi_{xx}(t) = \gamma(t)G, \qquad G = 1 + 2\langle n\rangle_{\mathrm{th}}. \tag{4}$$

The symbols ω and γ mean the frequency and damping coefficient normalized by the initial frequency ω_i, and $\langle n\rangle_{\mathrm{th}}$ is the equilibrium mean number of photons for the selected mode. The choice of coefficients in (2) and (4) was explained and justified in [16].

The solution of the system of equations (2) under condition (3) can be expressed in terms of the function $\varepsilon(t)$, which satisfies the *classical equation of motion* of the harmonic oscillator with time-dependent frequency

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0 \tag{5}$$

and the initial condition $\varepsilon(t) = \exp(-\mathrm{i}t)$ for $t \to -\infty$. Note that $\varepsilon(t)$ does not depend on the damping coefficient $\gamma(t)$. This coefficient enters the solutions $\hat{x}(t)$ and $\hat{p}(t)$ through the integral $\Gamma(t) = \int_{-\infty}^{t}\gamma(\tau)\,\mathrm{d}\tau$. It can be shown [16] that if initially (at $t \to -\infty$) the field mode was in the thermal state, then at the moment $t > 0$ the mean number of photons equals

$$\mathcal{N}(t) = G\,\mathrm{e}^{-2\Gamma(t)}\left\{\frac{1}{2}E(t) + \int_{-\infty}^{t}\mathrm{d}\tau\,\mathrm{e}^{2\Gamma(\tau)}\gamma(\tau)(E(t)E(\tau) - \mathrm{Re}[\tilde{E}^*(t)\tilde{E}(\tau)])\right\} - \frac{1}{2}, \tag{6}$$

where

$$E(\tau) = \tfrac{1}{2}[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2], \qquad \tilde{E}(\tau) = \tfrac{1}{2}[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)]. \tag{7}$$

Formula (6) is *exact* for arbitrary functions $\omega(t)$ and $\gamma(t)$. However, we are interested here in the special case when the functions $\omega(t)$ and $\gamma(t)$ have the form of *periodical* pulses, separated by intervals of time with $\omega = 1$ and $\gamma = 0$ (we neglect the damping of the field between pulses, supposing that the quality factor of the cavity is big enough). Since the relative change of the frequency $\omega(t)$ during pulses is very small, the functions $E(\tau)$ and $\tilde{E}(\tau)$ can be considered as *approximate integrals of motion* during each pulse. Then after n pulses, we have [16]

$$\mathcal{N}_n = \frac{G}{2}\,\mathrm{e}^{-2\Lambda n}\left\{E_n + (1-\mathrm{e}^{-2\Lambda})\sum_{k=1}^{n}\mathrm{e}^{2\Lambda k}[E_nE_k - \mathrm{Re}(\tilde{E}_n^*\tilde{E}_k)]\right\} - \frac{1}{2}, \tag{8}$$

where $\Lambda = \int\gamma(\tau)\,\mathrm{d}\tau$, the integral being taken over the duration of a *single* pulse. The quantities E_k and $\tilde{E}_k$ mean the values of $E(\tau)$ and $\tilde{E}(\tau)$ at the end of the kth pulse. In the interval between the kth and $(k+1)$th pulses the function $\varepsilon(t)$ can be written as $\varepsilon_k(t) = a_k\,\mathrm{e}^{-\mathrm{i}t} + b_k\,\mathrm{e}^{\mathrm{i}t}$ (with $a_0 = 1$ and $b_0 = 0$), so that $E_k = |a_k|^2 + |b_k|^2$ and $\tilde{E}_k = 2a_kb_k$. Now we notice that if $\varepsilon(t) = \mathrm{e}^{-\mathrm{i}t}$ before the 'effective potential barrier' $V_{\mathrm{eff}}(t) = \omega^2(t) - 1$, then after the 'barrier' the solution can be written as $\varepsilon(t) = \rho_-\,\mathrm{e}^{-\mathrm{i}t} + \rho_+\,\mathrm{e}^{\mathrm{i}t}$, so that the ratio $r = \rho_+/\rho_-$ can be interpreted as the effective complex amplitude-reflection coefficient, whereas the complex number $\rho_- = f$ has the meaning of the *inverse* amplitude-transmission coefficient. An important additional parameter is the periodicity of pulses T (or the related phase shift $\theta = \omega_0 T$ in the dimensional units). All complex coefficients a_k and b_k can be easily expressed in terms of the complex parameters r and $f \equiv |f|\exp(\mathrm{i}\varphi)$ and real parameter T with the aid of the transfer matrix method [3, 18]

$$a_n = f\frac{\sinh(n\nu)}{\sinh(\nu)}\,\mathrm{e}^{-\mathrm{i}T(n-1)} - \frac{\sinh[(n-1)\nu]}{\sinh(\nu)}\,\mathrm{e}^{-\mathrm{i}Tn}, \qquad b_n = rf\frac{\sinh(n\nu)}{\sinh(\nu)}\,\mathrm{e}^{\mathrm{i}T(n-1)}, \tag{9}$$

where the new coefficient ν is determined by the relations

$$\cosh(\nu) = |\mathrm{Re}[f \exp(\mathrm{i}T)]| \equiv |f \cos(\delta)|, \qquad \delta = T - T_{\mathrm{res}}, \quad T_{\mathrm{res}} = m\pi - \varphi. \tag{10}$$

Then the summation in equation (8) is reduced to calculating several finite geometric progressions. After some algebra one can arrive at the following formula for the mean number of quanta created after n pulses (we assume that coefficient ν is *real*):

$$\mathcal{N}_n = \frac{G|rf|^2 \exp(-\Lambda)}{4\sinh(\nu)} \left[\frac{\exp[2n(\nu-\Lambda)]}{\sinh(\nu-\Lambda)} + \frac{\exp[-2n(\nu+\Lambda)]}{\sinh(\nu+\Lambda)} \right] + \frac{G-1}{2} - \frac{G|rf|^2[1+\exp(-2\Lambda)]}{4\sinh(\nu-\Lambda)\sinh(\nu+\Lambda)}. \tag{11}$$

We see that photons can be generated if $\nu > \Lambda$. Under realistic experimental conditions, the parameters $|r|$ and Λ are very small. Moreover, the parameter ν can be real only if the *detuning coefficient* δ is also small. Then one finds from (10) that $\nu \approx \sqrt{|r|^2 - \delta^2}$. Formula (11) can be simplified in the asymptotical regime $n(\nu - \Lambda) \gg 1$

$$\mathcal{N}_n \approx \frac{G|r|^2}{4\nu(\nu-\Lambda)} \exp[2n(\nu-\Lambda)] + \frac{G-1}{2}. \tag{12}$$

(we assume that the difference $(\nu - \Lambda)$ is not too close to zero). The rate of photon generation is maximal for $\delta = 0$, i.e. under the condition of *exact resonance*, when the periodicity of pulses T coincides with T_{res} (only this case was considered in [3, 16]). In the dimensional variables, we have

$$T_{\mathrm{res}} = \tfrac{1}{2} T_0 (m - \varphi/\pi), \tag{13}$$

where T_0 is the period of oscillations in the field mode. There is no photon generation if the detuning coefficient exceeds the critical value $\delta_{\max} = \sqrt{|r|^2 - \Lambda^2}$.

3. Time-dependent shift of effective complex frequency: TE versus TM modes

For small variations of the effective frequency $\omega(t)$, we can write $\omega(t) = \omega_0[1 + \chi(t)]$ with $|\chi| \ll 1$. Then using the formulae from [19], we can express the absolute value of the single-barrier amplitude-reflection coefficient $|r|$ and the phase φ of the single-barrier inverse transmission coefficient f as [20] (here we return to the dimensional variables)

$$|r| \approx \left| \int_{t_i}^{t_f} \omega_0 \chi(t)\, \mathrm{e}^{-2\mathrm{i}\omega_0 t}\, \mathrm{d}t \right|, \qquad \varphi \approx \omega_0 \int_{t_i}^{t_f} \chi(t)\, \mathrm{d}t, \tag{14}$$

where t_i and t_f correspond to the beginning and end of the pulse. For the harmonic oscillations at the double frequency $\chi(t) = \chi_0 \sin(2\omega_0 t)$ with $t_i = 0$ and $t_f = \pi/\omega_0$, we have $\varphi \equiv 0$. Then (13) becomes the usual parametric resonance condition $T = T_0/2$. In this case, equation (12) coincides with the result obtained in [9]. For other shapes of the function $\chi(t)$, some fine tuning of the period between pulses T must be made, taking into account the concrete value of the phase shift φ.

To calculate the values of the parameters $|r|$ and φ which can be obtained in the case of a semiconductor mirror, we consider a cylindrical cavity with an arbitrary cross section and the axis parallel to the x-direction. We assume that the main part of the cavity is empty: $\varepsilon(x) \equiv 1$ for $-L < x < 0$, but there is a thin slab of a semiconductor material with $\varepsilon(x) \neq 1$ in the region $0 < x < D \ll L$. We start from the Maxwell equations, which do not contain derivatives of function $\varepsilon(\mathbf{r}, t)$ with respect to the time variable: $\mathrm{rot}\,\mathbf{B} = \partial\mathbf{D}/(c\partial t)$ and $\mathrm{rot}(\mathbf{D}/\varepsilon) = -\partial\mathbf{B}/(c\partial t)$ (we consider non-magnetic media). Excluding the magnetic field $\mathbf{B}$, we obtain the equation for the monochromatic field $\mathbf{D}(\mathbf{r}, t) = \mathbf{D}(\mathbf{r})\exp(-\mathrm{i}\Omega t)$

$$\mathrm{grad}\,\mathrm{div}(\mathbf{D}/\varepsilon) - \Delta(\mathbf{D}/\varepsilon) = (\Omega^2/c^2)\mathbf{D}. \tag{15}$$

For TE modes, vectors $\mathbf{E}$ and $\mathbf{D}$ are perpendicular to the x axis (and parallel to plane surfaces of the cylinder and slab). If we assume that the dielectric permeability depends only on the longitudinal space variable x, then $\mathrm{div}(\mathbf{D}/\varepsilon) = [\varepsilon(x)]^{-1}\,\mathrm{div}\,\mathbf{D} \equiv 0$, because all differentiations are made with respect to the transversal coordinates. Thus, the electric field $\mathbf{E} = \mathbf{D}/\varepsilon(x)$ satisfies the Helmholtz equation $\Delta\mathbf{E} + (\Omega/c)^2\varepsilon(x)\mathbf{E} = 0$, which allows for a factorization of any scalar component $E(x, \mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$,

$$\psi'' + \left[(\Omega/c)^2\varepsilon(x) - k_\perp^2\right]\psi = 0, \qquad \Delta_\perp\Phi + k_\perp^2\Phi = 0. \tag{16}$$

The solution to (16) in the region $-L < x < 0$, satisfying the boundary condition $\psi(-L) = 0$, is $\psi(x) = F_1\sin[k(x + L)]$, where the constant coefficient k is related to the field eigenfrequency Ω and the corresponding wavelength in vacuum λ as

$$\Omega = c\left(k^2 + k_\perp^2\right)^{1/2}, \qquad \lambda = 2\pi\left(k^2 + k_\perp^2\right)^{-1/2}. \tag{17}$$

Since the electric field and its derivative in the x-direction must be continuous at the surface $x = 0$, the admissible values of the parameter k can be determined from the condition of the continuity of the logarithmic derivative of $\psi(x)$ at $x = 0$

$$\tan(kL) = k\psi_+(0; k)/\psi_+'(0; k), \tag{18}$$

where $\psi_+(x; k)$ is the solution of equation (16) in the region $0 < x < D$, satisfying the boundary condition $\psi_+(D) = 0$. In the generic case (18) is a complicated transcendental equation, which can be solved only numerically. But for a *thin* layer, $D \ll \lambda \sim L$, the value of k must be close to π/L (we consider the lowest mode in the cavity). Thus, we can write $k = (1 + \xi)\pi/L$ with $|\xi| \ll 1$ and replace $\tan(\pi\xi)$ in the left-hand side of (18) simply by $\pi\xi$. Moreover, in the first approximation we can identify k with π/L *in the right-hand side*. Introducing the dimensionless variable $\tilde{x} = x/D$ (so that $0 \leqslant \tilde{x} \leqslant 1$) and the function $R(\tilde{x}) = \psi_+(\tilde{x})/\psi_+'(\tilde{x})$, we find $\xi_{\mathrm{TE}} = R(0)\Delta\eta$, where

$$\eta = \lambda/(2L) < 1, \qquad \Delta = 2D/\lambda \ll 1 \tag{19}$$

and the function $R(\tilde{x})$ satisfies the first-order nonlinear *generalized Riccati equation*

$$\mathrm{d}R/\mathrm{d}\tilde{x} = 1 + \pi^2\Delta^2[\varepsilon(\tilde{x}) - 1 + \eta^2]R^2 \tag{20}$$

with the boundary condition $R(1) = 0$.

In the TM case, the electric induction vector $\mathbf{D}$ can be expressed as $\mathbf{D} = D_\parallel\mathbf{e}_1 + \mathbf{D}_\perp$. Using the equation $\mathrm{div}\,\mathbf{D} \equiv \partial D_\parallel/\partial x + \nabla_\perp\cdot\mathbf{D}_\perp = 0$, one has $\mathrm{div}(\mathbf{D}/\varepsilon) = D_\parallel\partial(1/\varepsilon)/\partial x$. Factorizing the longitudinal component as $D_\parallel = \psi(x)\Phi(\mathbf{r}_\perp)$, we obtain from (15)

$$\psi'' + \left[(\Omega/c)^2\varepsilon(x) - k_\perp^2\right]\psi - [\varepsilon'(x)/\varepsilon(x)]\psi' = 0, \qquad \Delta_\perp\Phi + k_\perp^2\Phi = 0. \tag{21}$$

The function $\varepsilon'(x)$ is the derivative of $\varepsilon(x)$. At the interface $x = 0$, we have the continuity conditions $D_\parallel(0-) = D_\parallel(0+)$ and $\varepsilon^{-1}\partial D_\parallel/\partial x|_{0-} = \varepsilon^{-1}\partial D_\parallel/\partial x|_{0+}$. At the metallic plane surfaces we have the boundary conditions $\partial D_\parallel/\partial x|_{x=-L,D} = 0$. Thus we have now $\psi(x) = F_1\cos[k(x + L)]$ for $-L < x < 0$, so that the parameter k should be determined from the equation $\tan(kL) = -\psi_+'(0; k)/[\psi_+(0; k)k\varepsilon(0+)]$. Writing again $k = \pi(1 + \xi)/L$ and using the same approximation as in the TE case, we can write the small shift of the wave number as $\xi_{\mathrm{TM}} = S(0)\Delta/\eta$, where the function $S(\tilde{x}) = -[(\pi\Delta)^2\varepsilon(\tilde{x})]^{-1}\psi'/\psi$ satisfies the generalized Riccati equation (in the domain $0 < \tilde{x} < 1$)

$$\mathrm{d}S/\mathrm{d}\tilde{x} = 1 - (1-\eta^2)/\varepsilon(\tilde{x}) + (\pi\Delta)^2\varepsilon(\tilde{x})S^2. \tag{22}$$

In semiconductors, the real part of the dielectric function ε_1 is of the order of 10 or bigger. Therefore, the functions $R(\tilde{x})$ and $S(\tilde{x})$ practically coincide, because they satisfy the same boundary condition $R(1) = S(1) = 0$, whereas two slightly different equations (20) and (22) can be replaced by the unique equation

$$\mathrm{d}R/\mathrm{d}\tilde{x} = 1 + (\pi\Delta)^2\varepsilon(\tilde{x})R^2, \tag{23}$$

if one neglects the term $(1-\eta^2)/\varepsilon(\tilde{x})$, whose relative contribution is much less than 0.1 (especially for excited semiconductors with large imaginary part of the dielectric function).

Due to equation (17), the small relative shift of the resonance frequency is related to the shift of the longitudinal wave number ξ as $\chi_\Omega \equiv [\Omega - \omega_0]/\omega_0 = \eta^2(\xi - \xi_0)$, where ξ_0 corresponds to the non-excited semiconductor slab. Therefore, the shifts of eigenfrequencies of the TE and TM modes differ only by the factor η^2

$$\chi = \eta^\kappa\Delta\,[R(0) - R_0(0)]\,, \qquad \kappa_{\mathrm{TM}} = 1, \quad \kappa_{\mathrm{TE}} = 3. \tag{24}$$

The value $R_0(0)$ corresponds to the non-excited semiconductor with $\varepsilon_2 = 0$. Since $\eta < 1$, the frequency shift and the photon generation rate for TM polarization are always bigger than for TE. This conclusion agrees with those of [17] (for an ideal time-dependent dielectric slab) and [11, 20] (for cavities with oscillating ideal metallic boundaries).

For an arbitrary function $\varepsilon(\tilde{x})$, equations (20) and (22) or their unified version (23) should be solved numerically. However, in the specific case of a photo-excited semiconductor with rapidly decreasing large imaginary part $\varepsilon_2(\tilde{x})$, we can obtain a simple approximate analytical solution. Let us consider equation (23). If $\varepsilon_2(\tilde{x}) = 0$ and $\varepsilon_1(\tilde{x}) = \mathrm{const}$, it has the exact solution $R_0(\tilde{x}) = \tan[a(\tilde{x}-1)]/a$, where $a = \pi\Delta\sqrt{\varepsilon_1}$. Since $\varepsilon_1 \sim 10$ for semiconductors, $a < 0.1$ if $\Delta \leqslant 0.01$. Consequently, $R_0(0) \approx -1$, with the error of the order of 0.01 or smaller. When the semiconductor slab is illuminated by the laser pulse, $\varepsilon(\tilde{x}) = \varepsilon_1 + \mathrm{i}\varepsilon_2(\tilde{x})$, where the imaginary part $\varepsilon_2(\tilde{x})$ can attain very high values, so that $\pi^2\Delta^2\varepsilon_2(\tilde{x}) \gg 1$ in some region $0 < \tilde{x} < h$ near the surface of the slab (otherwise we cannot simulate the displacement of the effective boundary). It is important that $h \ll 1$, because the carriers are created in the thin layer of the depth α^{-1}, where α is the absorption coefficient of the laser radiation. Obviously, to create an effective 'plasma mirror' one needs the material with $h \sim \alpha^{-1} \ll 1$. In the region $0 < \tilde{x} < h$, we can neglect the first term 1 in the right-hand side of equation (23), as well as the real part of the coefficient at R^2 (because ε_1 after the excitation of the semiconductor is practically the same as before it, as long as the plasma frequency of the carriers remains much smaller than optical frequencies). The simplified equation yields

$$\frac{1}{R(0)} - \frac{1}{R(h)} = \mathrm{i}(\pi\Delta)^2\int_0^h \varepsilon_2(\tilde{x})\,\mathrm{d}\tilde{x}. \tag{25}$$

On the other hand, in the region $h < \tilde{x} < 1$ the nonlinear term in (23) becomes insignificant, so that we can write $R(1) - R(h) = 1 - h$. Since $R(1) = 0$ and $h \ll 1$, we can take $R(h) = -1$. Moreover, since the function $\varepsilon_2(\tilde{x})$ quickly goes to zero outside the interval $(0, h)$, we can extend the upper limit of integration in (25) to infinity. Thus we have $R(0) = (\mathrm{i}A - 1)^{-1}$ and finally

$$\chi_\Omega \equiv \chi - \mathrm{i}\gamma = \eta^\kappa\Delta\frac{A^2 - \mathrm{i}A}{A^2+1}, \qquad \chi = \eta^\kappa\Delta\frac{A^2}{A^2+1}, \qquad \gamma = \eta^\kappa\Delta\frac{A}{A^2+1}, \tag{26}$$

where $\kappa = 3$ for the TE mode and $\kappa = 1$ for the TM mode. The coefficient A equals

$$A = (\pi\Delta)^2 \int_0^\infty \varepsilon_2(\tilde{x})\,\mathrm{d}\tilde{x}. \tag{27}$$

We have inspected the solution (26), solving the Riccati equation (23) numerically for two dielectric functions $\varepsilon(\tilde{x}) = 10 + \mathrm{i}B\exp(-\alpha\tilde{x})$ (in this case equation (16) can be solved analytically in terms of Bessel functions [20]) and $\varepsilon(\tilde{x}) = 10 + \mathrm{i}B\cosh^{-2}(\alpha\tilde{x})$ with different parameters B and α. The coefficient A (27) in both cases equals B/α. For $\alpha = 100$, the relative difference between the numerical solutions and analytical expressions in (26) was less than 0.001 for any value $B > 100$ (for both functions). A difference at the level of a few per cent was noticed only for values $\alpha < 10$. These results show that the approximation (26) is very good indeed.

The function $\varepsilon_2(\tilde{x})$ is proportional to the concentration of carriers inside the slab, which is governed by the equations, which take into account, besides the photo-absorption, different recombination processes. For highly doped semiconductors, when the trap-assisted recombination prevails over other recombination mechanisms, a good approximation is the *linear* equation [3]

$$\partial n/\partial t = \nabla\cdot(Y\nabla n) + (\alpha\zeta/E_g)I(t)\,\mathrm{e}^{-\alpha x} - \beta_1 n, \tag{28}$$

where Y is the coefficient of ambipolar diffusion, α is the absorption coefficient of the laser radiation inside the layer, E_g is the energy gap of the semiconductor (which is close to the energy of laser photons), $I(t)$ is the time-dependent intensity of the laser pulse which enters the slab, $\zeta \leqslant 1$ is the efficiency of the photo-electron conversion, and β_1 is the trap-assisted recombination coefficient. According to equations (26) and (27), we need only the time-dependent integral value $\mathcal{K}(t) = \int_0^\infty n(x,t)\,\mathrm{d}x$ (we consider the case of high absorption coefficient, where the integration over the slab can be extended to infinity with very small error). Integrating equation (28) over dx from 0 to ∞ and taking into account the boundary condition $Y(\partial n/\partial x)|_{x=0} = 0$ (this means that we neglect the surface recombination), we obtain the equation $\partial\mathcal{K}/\partial t = (\zeta/E_g)I(t) - \beta_1\mathcal{K}$, which can be integrated immediately: $\mathcal{K}(t) = (\zeta/E_g)\int_0^t \exp[-\beta_1(t-\tau)]I(\tau)\,\mathrm{d}\tau$. If the duration of the laser pulse is much less than the recombination time, then

$$\mathcal{K}(t) = \frac{\zeta W}{E_g S}\,\mathrm{e}^{-\beta_1 t}, \qquad W/S = \int_{-\infty}^{\infty} I(\tau)\,\mathrm{d}\tau, \tag{29}$$

where W is the total energy of the laser pulse and S is the area of the surface of the semiconductor slab (it is assumed that the energy is distributed uniformly over this area). Combining equations (27) and (29) with the relation $\sigma(x,t) = n(x,t)|eb|$, where e is the electron charge and b is the total mobility of carriers (for each electron–hole pair), we can write the time-dependent function $A(t)$ in equation (26) as

$$A(t) = A_0\,\mathrm{e}^{-\beta_1 t}, \qquad A_0 = 4\pi^2|eb|\zeta W\Delta/(cE_g S). \tag{30}$$

4. Discussion of numerical values and optimal parameters

According to equation (12), the rate of photon generation is determined by the difference $(\nu - \Lambda)$. Using the analytical expression for $\gamma(t)$ from (26) and (30), one can calculate the integral for Λ exactly (assuming that the period between pulses T is much bigger than the relaxation time $T_r = \beta_1^{-1}$ and extending the integration interval to infinity). Let us consider first the case of strict resonance (13), when $\nu = |r|$. Due to equations (14) and (26), both coefficients $|r|$ and Λ are proportional to the product $\eta^\kappa\Delta$. Therefore the number of photons generated after $n \gg 1$ pulses is proportional to $\exp[2n\eta^{\kappa}\Delta F(A_0, Z)]$, where $Z = \omega_0/\beta_1 = 2\pi T_r/T_0$ and

$$F(A_0, Z) = \frac{Z}{2}\left|\int_0^{\infty} \frac{A_0^2 \exp(-\mathrm{i}Z\tau)\,\mathrm{d}\tau}{A_0^2 + \exp(\tau)}\right| - Z\tan^{-1}(A_0). \qquad (31)$$

A detailed numerical analysis of the amplification coefficient $F(A_0, Z)$ was made in [3]. It appears that the generation of photons is absolutely impossible (i.e., $F < 0$) if $Z > 0.54$ or $A_0 < 4$. For moderate values of parameter A_0, the maximal values of F are achieved for $Z \approx 0.3$. This dimensionless value corresponds to the recombination time $T_r \approx 20$ ps for $T_0 = 400$ ps (or $f_0 = 2.5$ GHz). The parameter A_0 is proportional to the energy of the laser pulse W. If we fix the number of photons which should be created after n pulses, then $n \approx \mathrm{const}/F$. Consequently, the function $B_0(A_0, Z) = A_0/F(A_0, Z)$ is proportional to the *total energy* of all necessary laser pulses. Choosing for each value of A_0, the optimal value of the parameter Z (when $F(A_0, Z)$ attains its local maximum as a function of Z) we have found that the minimum of function $B_0(A_0, Z)$ happens at $A_{0*} \approx 10$, when $F_* = 0.18$, $|r_*| = 0.61\eta^{\kappa}\Delta$ and $\Lambda_* = 0.43\eta^{\kappa}\Delta$. We see that the ratio $\Lambda_*/|r_*| \approx 0.7$ is rather high, so that neglecting the effect of damping would result in a significant error. For the geometry reported in [15], i.e. $D = 0.6$ mm, $S = 7 \times 2$ cm, $\lambda = 12$ cm (or $f_0 = 2.5$ GHz), we obtain, taking in formula (30) a typical value of mobility $b = 3 \times 10^6$ CGS units $= 1\ \mathrm{m^2\ V^{-1}\ s^{-1}}$, $E_g = 1.4$ eV (as for GaAs) and $\zeta = 1$, that $A_0 = 10$ for $W \sim 2 \times 10^{-3}$ J. The resonance mode of the rectangular cavity described in [15] has TE polarization with $\Delta = 0.01$ and $\eta = 0.55$ ($L = 110$ mm), so that $\eta^3 = 0.16$. Then, putting in formula (12) $G = 1$ and optimal values of A_0 and Z, we find that $\mathcal{N}_3 = 10^3$ photons can be created after $n_3 \approx 12\,000$ pulses. Taking the periodicity of pulses $T \approx T_0/2 = 200$ ps, the total duration of the process must be $T_{\mathrm{tot}} \sim 2\ \mu$s, which means that the cavity quality factor must be higher than $Q_{\min} = \pi T_{\mathrm{tot}}/T_0 \approx \pi n/2 \sim 2 \times 10^4$, which can be easily achieved. The problem is that the total energy of all pulses must be about 20 J, and this value seems to be too high (because this energy will heat the semiconductor after the recombination, so it must be quickly removed). One way to diminish the energy of laser pulses is to search for semiconductors with higher mobility of carriers (because parameter A_0 is proportional to the product $W|b|$). The maximal mobility in GaAs, reported in the available literature, can exceed $100\ \mathrm{m^2\ V^{-1}\ s^{-1}}$. It is not clear, however, whether such big values are compatible with small recombination times, which are achieved by means of strong doping.

An optimization of geometry could also help. Indeed, the resonance wavelength λ corresponding to the mode TE_{101} of the rectangular cavity is related to the cavity length L and the biggest transverse dimension B as $\lambda = 2LB/\sqrt{L^2 + B^2}$. Consequently, $B = \lambda/(2\sqrt{1-\eta^2})$. For the fixed values of parameter A_0 and the smallest transverse dimension, the energy of the pulse is proportional to the area of the surface, i.e. B, whereas the necessary number of pulses depends on L as $\eta^{-3} \sim L^3$. Consequently, the total energy is proportional to the product BL^3. Minimizing this product for the fixed value of λ (which is equivalent to maximization of the function $\eta^3\sqrt{1-\eta^2}$), we find the optimal value $\eta_{\mathrm{opt}}^{\mathrm{TE}} = \sqrt{3}/2 = 0.866$, which corresponds to $L = \lambda/\sqrt{3} \approx 7$ cm and $B = \lambda = 12$ cm. This choice could diminish the total energy by 2.5 times and the number of pulses by almost four times, at the expense of increasing the energy of each pulse by approximately two times. Besides, one can increase the generation rate, increasing the thickness D of the slab (remember that $\Delta = 2D/\lambda$).

In cavities with *the same* geometry, the rate of generation is higher for the TM modes than for TE ones, because the frequency shift χ is proportional to the factor η instead of η^3. Unfortunately, there exist no TM modes whose eigenfrequencies depend only on one of two transverse dimensions. Therefore, the semiconductor slab must have *two* sides bigger than $\lambda/2 = 6$ cm. The optimal geometry in this case (TM_{111} mode) has a quadratic cross section (for rectangular cavities; the results for circular cross sections are practically the same), and the parameter η can be found from the maximization of the function $\eta(1-\eta^2)$, which yields $\eta_{opt}^{TM} = 1/\sqrt{3} \approx 0.58$. This value corresponds to a *cubical* cavity with the length $L = 104$ mm. The transverse area in such a case must exceed 100 cm^2, which is so great that the total energy of 3300 pulses, necessary to create 1000 photons, is five times bigger than for the optimized TE_{101} mode. Moreover, $\left(\eta_{opt}^{TE}\right)^3 / \eta_{opt}^{TM} = 9/8$, so the generation of the same number of photons in the optimized TE_{101} mode requires 10% fewer pulses than in the optimized TM_{111} mode. Note that we suppose that the *whole* surface of the slab is illuminated *uniformly*. This can be a difficult technical problem. Therefore, it would be extremely important to calculate the time-dependent complex frequency shift of the selected mode in the case of *nonuniform illumination*, in particular, when only some central part of the slab is illuminated.

The numerical values of the necessary number of pulses and the total energy, given above, may seem disappointing. Note, however, that they correspond to the case of the initial *vacuum state*. As a matter of fact, preparing the field mode in the initial vacuum state can be a nontrivial problem in itself, because the equilibrium mean number of quanta with frequency $\nu = 2.5$ GHz at the helium temperature $\Theta = 4$ K equals $\langle n\rangle_{th} \approx k_B\Theta/h\nu \approx 35$ (we take into account that $k_B\Theta \gg h\nu$). On the other hand, the parametric amplification of the number of photons from the initial *thermal state* can be an easier task [10]. Indeed, in this case the G-factor in formula (12) equals $G = 70$, and this high initial value reduces the necessary number of pulses (for $\mathcal{N}_3 = 1000$) to $n_3^{th} \approx 5000$ for $\eta = 0.55$ and to $n_3^{th} \approx 1200$ for the optimized cavity with $\eta = 0.866$. The total energy in the optimal case can be reduced to about 3 J. Moreover, one could try first to work at higher temperatures. For example, taking $\Theta = 20$ K (when Nb is still a superconductor, so that the cavity does not lose its high quality factor) we have $\langle n\rangle_{th} \approx 170$. Then $\mathcal{N}_3 = 10^3$ photons can be generated after only about 500 pulses for the optimized geometry. Another possible advantage of working at higher temperatures could be higher mobility of carriers: due to the low-temperature dependence $b \sim \Theta^{+3/2}$, the increase of temperature from 4 K to 20 K can increase the mobility by ten times.

The level of $\mathcal{N}_3 = 10^3$ generated photons corresponds to the presumed sensitivity of the measuring apparatus, and it is significantly bigger than the mean number of thermal photons and their fluctuations in the selected cavity mode in the examples considered. Note that the total energy of laser pulses of the order of a few Joules corresponds to about 10^{20} photons entering the cavity. Although the main part of them will be absorbed by the semiconductor slab, some portion will be reflected from the surface. Therefore, one could worry about the noise from these laser photons. However, they have frequencies in the visible or near infrared band, whereas the measuring apparatus is supposed to be sensitive only to the frequencies near the resonance cavity frequency (2.5 GHz). These five orders of magnitude separating the frequencies of laser and 'Casimir' photons, seem to be a safe guarantee of the absence of additional laser noise. Besides, since the laser light is supposed to enter the cavity through a thin optical fibre, no photons with wave length exceeding a few millimetres (the diameter of the fibre) will be transmitted through this optical waveguide.

However, we would like to emphasize once more time the importance of taking into account the correct value of the phase shift φ in the resonance conditions (10) and (13). It can be calculated exactly from equations (14), (26) and (30): $\varphi = (Z/2)\eta^{\kappa}\Delta \ln\left(1 + A_0^2\right)$. Thus for $A_0 = 10$ and $Z = 0.3$, we have $\varphi \approx 0.7\eta^{\kappa}\Delta$. Under the same conditions, the critical detuning equals $\delta_{max} \approx 0.7|r| \approx 0.4\eta^{\kappa}\Delta \approx \varphi/2$. Consequently, taking $T = T_0/2$ exactly, one certainly will be out of the resonance, and no photons will be generated! On the other hand, one could use the off-resonance sequence of pulses in order to measure the noise level.

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